$$\frac{du}{dC_1}=\frac{C}{1+C^2C_1^2},\quad \frac{du}{dC_2}=-\frac{C}{1+C^2C_2^2},$$

$$\frac{d^2u}{dC_1^2}=-\frac{2C^3C_1}{(1+C^2C_1^2)^2},\quad \frac{d^2u}{dC_2^2}=\frac{2C^3C_2}{(1+C^2C_2^2)^2},\quad \frac{d^2u}{dC_1dC_2}=0.$$

Now $\dfrac{d^2u}{dC_1^2}\dfrac{d^2u}{dC_2^2}<\dfrac{d^2u}{dC_1dC_2}$.

Hence there is no maximum unless $C_1=\infty$, $C_2=-\infty$.

II. Solution by V. M. SPUNAR, Chicago, Ill., and the PROPOSER.

By the principle of the instrument, we have

$$\frac{\tan\phi_1}{\tan\phi_2}=\frac{c_1}{c_2}.\quad \therefore \frac{\tan\phi_1-\tan\phi_2}{\tan\phi_1+\tan\phi_2}=\frac{c_1-c_2}{c_1+c_2}\ldots(1).$$

But by trigonometry,

$$\tan\phi_1-\tan\phi_2=\frac{\sin(\phi_1-\phi_2)}{\cos\phi_1\cos\phi_2},\quad \tan\phi_1+\tan\phi_2=\frac{\sin(\phi_1+\phi_2)}{\cos\phi_1\cos\phi_2}.$$

Hence (1) becomes

$$\frac{\sin(\phi_1-\phi_2)}{\sin(\phi_1+\phi_2)}=\frac{c_1-c_2}{c_1+c_2},\text{ or }\sin(\phi_1-\phi_2)=\left(\frac{c_1-c_2}{c_1+c_2}\right)\sin(\phi_1+\phi_2).$$

Hence, $(\phi_1-\phi_2)$ is a maximum when $\sin(\phi_1+\phi_2)$ is maximum; that is, when $\sin(\phi_1+\phi_2)=1$; that is, $\phi_1+\phi_2=\frac{1}{2}\pi$.

297. Proposed by PROF. L. C. WALKER, Socorro, New Mexico.

A square hole $2s$ on a side is cut through an ellipsoid, axes $2a$, $2b$, $2c$, the axis of the hole coinciding with the axis $2c$ of the ellipsoid. Find (1) the volume, and (2) the surface removed.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

The volume $V=8c\displaystyle\int_0^s dx\int_0^s dy\sqrt{1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2}$

$$=\frac{4c}{b}\left[\frac{bs}{a}\int_0^s\sqrt{\frac{a^2(b^2-s^2)}{b^2}-x^2}+\frac{b^2}{a^2}\int_0^s(a^2-x^2)\sin^{-1}\frac{as}{b\sqrt{(a^2-x^2)}}\partial x\right.$$

$$=\frac{4bcs}{3a^2}(3a^2-s^2)\sin^{-1}\frac{as}{b\sqrt{(a^2-s^2)}}+\frac{4acs(3b^2-s^2)}{3b^2}\sin^{-1}\frac{bs}{a\sqrt{(b^2-s^2)}}$$

$$+\tfrac{8}{3}abc\sin^{-1}\frac{s^2}{(a^2-s^2)^{\frac{1}{2}}(b^2-s^2)^{\frac{1}{2}}}+\frac{10cs^2}{3ab}\sqrt{[a^2b^2-(a^2+b^2)s^2]}.$$

The surface $S=\dfrac{4}{ab}\displaystyle\int_0^s dx\int_0^s dy\sqrt{\frac{a^4b^4-b^4(a^2-c^2)x^2-a^4(b^2-c^2)y^2}{a^2b^2-b^2x^2-a^2y^2}}$.

The integration of this leads to elliptical functions.

NOTE. If any one will complete the solution of the second part of this problem we shall be pleased to publish it. ED. F.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

NOTE. Dr. Whalen, of the University of Illinois, has consented to edit this department in the future. Will contributors please send their problems and solutions to him?

174. Proposed by B. KRAMER, Student, University of Pittsburg, Pittsburg, Pa.

Find a general solution of $x(x+a)=y^2$, a, x, and y being integers. Given a, required to find x to satisfy conditions.

I. Solution by PROFESSOR F. L. GRIFFIN, Ph. D., Williams College.

Let d be any integer contained in a an odd number of times. Then the general solution is: $x=(a-d)^2/4d$, $y=(a^2-d^2)/4d$.

For let $y=x+n$, whence $x^2+ax=x^2+2nx+n^2$, or $x=n^2/(a-2n)$.

Now x is integral if, and only if, $a-2n$ is a factor of n; that is, if an integer k exists such that $n=kd$ [denoting $a-2n$ by d]. But then $a=d+2n=d(1+2k)$; so that integral values of x exist if, and only if, d is contained in a an odd number of times.

Using any such number d we have, $k=(a-d)/2d$, $n=\frac{1}{2}(a-d)$, $x=(a-d)^2/4d$, $y=x+\frac{1}{2}(a-d)=(a^2-d^2)/4d$, as stated.

Remarks. (I) Negative values of d are admissible, as the factors x and $x+a$ merely have their numerical values interchanged.

(II) If a be prime, $d=\pm1$ gives the only solution except the trivial case $d=a$, $x=y=0$.

(III) Numerical examples follow: